**SC531 – Lecture #10**

**JOINT DISTRIBUTION, COVARIANCE, CORRELATION**

Definition (Ref. #2):

Let S be the sample space of an experiment, such that each trial yields an outcome s in S. Let X(s) and Y(s) be two numbers associated with each possible outcome s of a trial. Then the set of pairs:

{ ( X(s), Y(s) ) | s in S }

defines a two-dimensional random number.

The probability mass function [or probability density function], of this two-dimensional random variable is called a *joint probability mass* *function* [or *joint probability density function*].

Note that each trial can yield, in general, a set of numbers, but we will consider only two. And we will consider only discrete RVs here.

Simple examples: rolling two dice, dentist’s clinic.

**A trivial example of a joint distribution:**

Throwing two dice, but with the number thrown by each die considered as being separate. Note that now each trial yields a pair of numbers – as long as you treat them as separate numbers!

[Please note that, if we **choose** to add up the two numbers, we get a single RV with range 2 to 12.]

Under normal circumstances, the joint distribution would be:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Y** | | | | | | |
|  |  | **1** | **2** | **3** | **4** | **5** | **6** |
| **X** | **1** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| **2** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| **3** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| **4** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| **5** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |
| **6** | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 | 1/36 |

Now suppose you come across a pair of dice which behaved like this:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Y | | | | | | |
|  |  | **1** | **2** | **3** | **4** | **5** | **6** |
| X | **1** | 1/6 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 1/6 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 1/6 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 1/6 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 1/6 | 0 |
| **6** | 0 | 0 | 0 | 0 | 0 | 1/6 |

This describes a strange situation, does it not?

Would you consider this as TWO dice or ONE? If two dice behave as one, there is some interesting physics or engineering at work, right?

In this second case, we say that the numbers thrown up by the two dice are totally *correlated*. In practice, correlation is usually not total, but partial. Degree of correlation has be calculated.

The original pair of dice had, as we expect, uncorrelated outputs.

Covariance and correlation – for two (discrete) RVs:

Cov(X,Y) = **Exp**[(X-mX)(Y-mY)] = Si (Xi-mX)(Yi-mY)prob(xi,yi)



Note: Let X be a random variable, and let g(X) be a function of X. Then the expected value of g(X) is defined as Si g(xi)prob(xi), and it is often denoted by Exp[f(X)]. Then mX is nothing but Exp[X].

Recall that earlier we went from variance to standard deviation.

In a somewhat similar way, the covariance of two RVs can be reduced to the dimensionless correlation coefficient, defined as:

rXY = Cov(X,Y)/ sXsY

In the two examples above, the two correlation coefficients will be, respectively, 0 and 1. You can verify this.

In general, the correlation coefficient of two RVs lies between -1 and 1.

Scatter diagrams can be used to visualise correlation.

Important: Difference between correlation and causality.